75. This limit has the form $\infty \cdot 0$.

$$\lim_{x\to -\infty}(x^2 - x^3)e^{2x} = \lim_{x\to -\infty}\frac{x^2 - x^3}{e^{-2x}} \quad \left[\tfrac{\infty}{\infty}\text{ form}\right] \quad \overset{\text{H}}{=} \lim_{x\to -\infty}\frac{2x - 3x^2}{-2e^{-2x}} \quad \left[\tfrac{\infty}{\infty}\text{ form}\right]$$

$$\overset{\text{H}}{=} \lim_{x\to -\infty}\frac{2 - 6x}{4e^{-2x}} \quad \left[\tfrac{\infty}{\infty}\text{ form}\right] \quad \overset{\text{H}}{=} \lim_{x\to -\infty}\frac{-6}{-8e^{-2x}} = 0$$

76. This limit has the form $0 \cdot (-\infty)$. $\displaystyle\lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+}\frac{\ln x}{1/x^2} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{1/x}{-2/x^3} = \lim_{x\to 0^+}\left(-\tfrac{1}{2}x^2\right) = 0$

77. This limit has the form $\infty - \infty$.

$$\lim_{x\to 1^+}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1^+}\left(\frac{x\ln x - x + 1}{(x-1)\ln x}\right) \overset{\text{H}}{=} \lim_{x\to 1^+}\frac{x\cdot(1/x) + \ln x - 1}{(x-1)\cdot(1/x) + \ln x} = \lim_{x\to 1^+}\frac{\ln x}{1 - 1/x + \ln x}$$

$$\overset{\text{H}}{=} \lim_{x\to 1^+}\frac{1/x}{1/x^2 + 1/x} = \frac{1}{1+1} = \frac{1}{2}$$

78. $y = (\tan x)^{\cos x} \quad \Rightarrow \quad \ln y = \cos x \, \ln \tan x$, so

$$\lim_{x\to(\pi/2)^-}\ln y = \lim_{x\to(\pi/2)^-}\frac{\ln\tan x}{\sec x} \overset{\text{H}}{=} \lim_{x\to(\pi/2)^-}\frac{(1/\tan x)\sec^2 x}{\sec x\,\tan x} = \lim_{x\to(\pi/2)^-}\frac{\sec x}{\tan^2 x} = \lim_{x\to(\pi/2)^-}\frac{\cos x}{\sin^2 x} = \frac{0}{1^2} = 0,$$

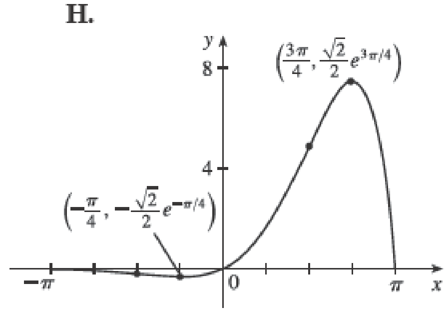
so $\displaystyle\lim_{x\to(\pi/2)^-}(\tan x)^{\cos x} = \lim_{x\to(\pi/2)^-} e^{\ln y} = e^0 = 1.$

79. $y = f(x) = e^x \sin x, -\pi \le x \le \pi$ **A.** $D = [-\pi, \pi]$ **B.** y-intercept: $f(0) = 0$; $f(x) = 0 \iff \sin x = 0 \Rightarrow x = -\pi, 0, \pi$. **C.** No symmetry **D.** No asymptote **E.** $f'(x) = e^x\cos x + \sin x\cdot e^x = e^x(\cos x + \sin x)$. $f'(x) = 0 \iff -\cos x = \sin x \iff -1 = \tan x \Rightarrow x = -\frac{\pi}{4}, \frac{3\pi}{4}$. $f'(x) > 0$ for $-\frac{\pi}{4} < x < \frac{3\pi}{4}$ and $f'(x) < 0$ for $-\pi < x < -\frac{\pi}{4}$ and $\frac{3\pi}{4} < x < \pi$, so f is increasing on $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and f is decreasing on $\left(-\pi, -\frac{\pi}{4}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$.

F. Local minimum value $f\left(-\frac{\pi}{4}\right) = (-\sqrt{2}/2)e^{-\pi/4} \approx -0.32$ and local maximum value $f\left(\frac{3\pi}{4}\right) = \left(\sqrt{2}/2\right)e^{3\pi/4} \approx 7.46$

G. $f''(x) = e^x(-\sin x + \cos x) + (\cos x + \sin x)e^x = e^x(2\cos x) > 0 \Rightarrow -\frac{\pi}{2} < x < \frac{\pi}{2}$ and $f''(x) < 0 \Rightarrow -\pi < x < -\frac{\pi}{2}$ and $\frac{\pi}{2} < x < \pi$, so f is CU on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and f is CD on $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$. There are inflection points at $\left(-\frac{\pi}{2}, -e^{-\pi/2}\right)$ and $\left(\frac{\pi}{2}, e^{\pi/2}\right)$.

H.



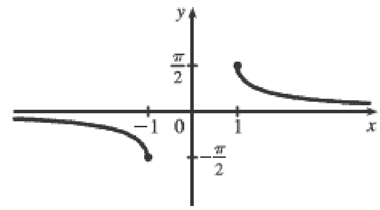
80. $y = f(x) = \sin^{-1}(1/x)$ **A.** $D = \{x \mid -1 \le 1/x \le 1\} = (-\infty, -1] \cup [1, \infty)$. **B.** No intercept
C. $f(-x) = -f(x)$, symmetric about the origin **D.** $\lim_{x \to \pm\infty} \sin^{-1}(1/x) = \sin^{-1}(0) = 0$, so $y = 0$ is a HA.

E. $f'(x) = \dfrac{1}{\sqrt{1-(1/x)^2}}\left(-\dfrac{1}{x^2}\right) = \dfrac{-1}{\sqrt{x^4 - x^2}} < 0$, so f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

F. No local extreme value, but $f(1) = \frac{\pi}{2}$ is the absolute maximum value and $f(-1) = -\frac{\pi}{2}$ is the absolute minimum value.

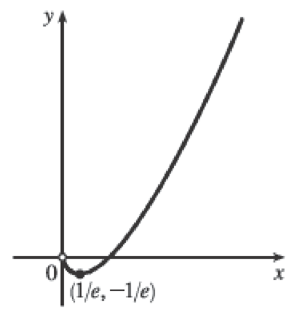
G. $f''(x) = \dfrac{4x^3 - 2x}{2(x^4 - x^2)^{3/2}} = \dfrac{x(2x^2 - 1)}{(x^4 - x^2)^{3/2}} > 0$ for $x > 1$ and $f''(x) < 0$ for $x < -1$, so f is CU on $(1, \infty)$ and CD on $(-\infty, -1)$. No IP

H.



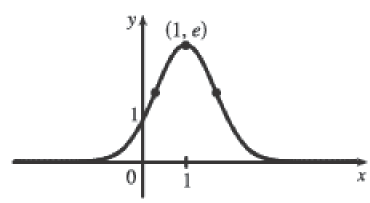
81. $y = f(x) = x \ln x$ **A.** $D = (0, \infty)$ **B.** No y-intercept; x-intercept 1. **C.** No symmetry **D.** No asymptote [Note that the graph approaches the point $(0, 0)$ as $x \to 0^+$.]
E. $f'(x) = x(1/x) + (\ln x)(1) = 1 + \ln x$, so $f'(x) \to -\infty$ as $x \to 0^+$ and $f'(x) \to \infty$ as $x \to \infty$. $f'(x) = 0 \Leftrightarrow \ln x = -1 \Leftrightarrow x = e^{-1} = 1/e$. $f'(x) > 0$ for $x > 1/e$, so f is decreasing on $(0, 1/e)$ and increasing on $(1/e, \infty)$. **F.** Local minimum: $f(1/e) = -1/e$. No local maximum.
G. $f''(x) = 1/x$, so $f''(x) > 0$ for $x > 0$. The graph is CU on $(0, \infty)$ and there is no IP.

H.



82. $y = f(x) = e^{2x - x^2}$ **A.** $D = \mathbb{R}$ **B.** y-intercept 1; no x-intercept **C.** No symmetry **D.** $\lim_{x \to \pm\infty} e^{2x - x^2} = 0$, so $y = 0$ is a HA. **E.** $y = f(x) = e^{2x - x^2} \Rightarrow f'(x) = 2(1 - x)e^{2x - x^2} > 0 \Leftrightarrow x < 1$, so f is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$. **F.** $f(1) = e$ is a local and absolute maximum value.
G. $f''(x) = 2(2x^2 - 4x + 1)e^{2x - x^2} = 0 \Leftrightarrow x = 1 \pm \frac{\sqrt{2}}{2}$.
$f''(x) > 0 \Leftrightarrow x < 1 - \frac{\sqrt{2}}{2}$ or $x > 1 + \frac{\sqrt{2}}{2}$, so f is CU on $\left(-\infty, 1 - \frac{\sqrt{2}}{2}\right)$ and $\left(1 + \frac{\sqrt{2}}{2}, \infty\right)$, and CD on $\left(1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right)$. IP at $\left(1 \pm \frac{\sqrt{2}}{2}, \sqrt{e}\right)$

H.



83. $y = f(x) = (x-2)e^{-x}$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = -2$; x-intercept: $f(x) = 0 \Leftrightarrow x = 2$

C. No symmetry **D.** $\lim_{x\to\infty} \frac{x-2}{e^x} \overset{\text{H}}{=} \lim_{x\to\infty} \frac{1}{e^x} = 0$, so $y = 0$ is a HA. No VA

E. $f'(x) = (x-2)(-e^{-x}) + e^{-x}(1) = e^{-x}[-(x-2)+1] = (3-x)e^{-x}$.

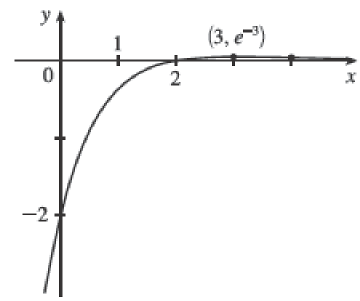
$f'(x) > 0$ for $x < 3$, so f is increasing on $(-\infty, 3)$ and decreasing on $(3, \infty)$.

F. Local maximum value $f(3) = e^{-3}$, no local minimum value

G. $f''(x) = (3-x)(-e^{-x}) + e^{-x}(-1) = e^{-x}[-(3-x)+(-1)]$
$= (x-4)e^{-x} > 0$

for $x > 4$, so f is CU on $(4, \infty)$ and CD on $(-\infty, 4)$. IP at $(4, 2e^{-4})$

H.



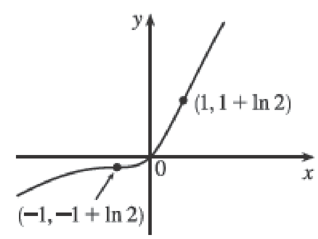
84. $y = f(x) = x + \ln(x^2+1)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0 + \ln 1 = 0$; x-intercept: $f(x) = 0 \Leftrightarrow$
$\ln(x^2+1) = -x \Leftrightarrow x^2 + 1 = e^{-x} \Rightarrow x = 0$ since the graphs of $y = x^2+1$ and $y = e^{-x}$ intersect only at $x = 0$.

C. No symmetry **D.** No asymptote **E.** $f'(x) = 1 + \frac{2x}{x^2+1} = \frac{x^2+2x+1}{x^2+1} = \frac{(x+1)^2}{x^2+1}$. $f'(x) > 0$ if $x \neq -1$ and f is increasing on $\mathbb{R}$. **F.** No local extreme values

G. $f''(x) = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2} = \frac{2[(x^2+1)-2x^2]}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+1)^2}$.

$f''(x) > 0 \Leftrightarrow -1 < x < 1$ and $f''(x) < 0 \Leftrightarrow x < -1$ or $x > 1$, so f is CU on $(-1, 1)$ and f is CD on $(-\infty, -1)$ and $(1, \infty)$. IP at $(-1, -1+\ln 2)$ and $(1, 1+\ln 2)$

H.

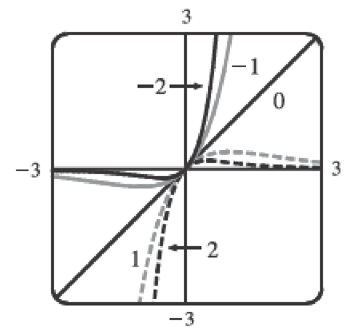


85. If $c < 0$, then $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} xe^{-cx} = \lim\limits_{x \to -\infty} \dfrac{x}{e^{cx}} \overset{\text{H}}{=} \lim\limits_{x \to -\infty} \dfrac{1}{ce^{cx}} = 0$, and $\lim\limits_{x \to \infty} f(x) = \infty$.

If $c > 0$, then $\lim\limits_{x \to -\infty} f(x) = -\infty$, and $\lim\limits_{x \to \infty} f(x) \overset{\text{H}}{=} \lim\limits_{x \to \infty} \dfrac{1}{ce^{cx}} = 0$.

If $c = 0$, then $f(x) = x$, so $\lim\limits_{x \to \pm\infty} f(x) = \pm\infty$, respectively.

So we see that $c = 0$ is a transitional value. We now exclude the case $c = 0$, since we know how the function behaves in that case. To find the maxima and minima of f, we differentiate: $f(x) = xe^{-cx} \quad \Rightarrow$ $f'(x) = x(-ce^{-cx}) + e^{-cx} = (1 - cx)e^{-cx}$. This is 0 when $1 - cx = 0 \quad \Leftrightarrow \quad x = 1/c$. If $c < 0$ then this represents a minimum value of $f(1/c) = 1/(ce)$, since $f'(x)$ changes from negative to positive at $x = 1/c$; and if $c > 0$, it represents a maximum value. As $|c|$ increases, the maximum or minimum point gets closer to the origin. To find the inflection points, we differentiate again: $f'(x) = e^{-cx}(1 - cx) \quad \Rightarrow$ $f''(x) = e^{-cx}(-c) + (1 - cx)(-ce^{-cx}) = (cx - 2)ce^{-cx}$. This changes sign when $cx - 2 = 0 \quad \Leftrightarrow \quad x = 2/c$. So as $|c|$ increases, the points of inflection get closer to the origin.

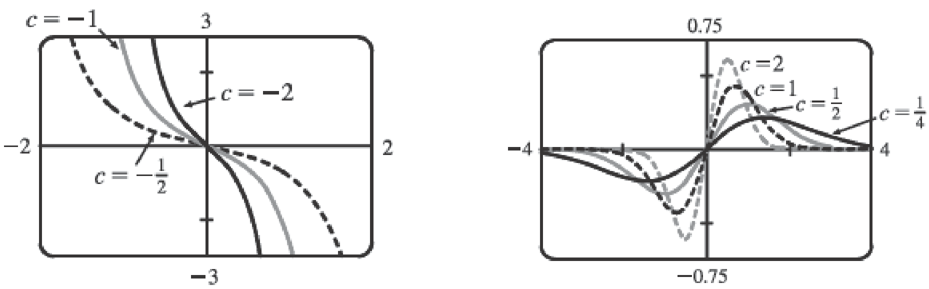


86.
We exclude the case $c = 0$, since in that case $f(x) = 0$ for all x. To find the maxima and minima, we differentiate:

$$f(x) = cxe^{-cx^2} \quad \Rightarrow \quad f'(x) = c\left[xe^{-cx^2}(-2cx) + e^{-cx^2}(1)\right] = ce^{-cx^2}(-2cx^2 + 1)$$

This is 0 where $-2cx^2 + 1 = 0 \quad \Leftrightarrow \quad x = \pm 1/\sqrt{2c}$. So if $c > 0$, there are two maxima or minima, whose x-coordinates approach 0 as c increases. The negative root gives a minimum and the positive root gives a maximum, by the First Derivative Test. By substituting back into the equation, we see that $f\left(\pm 1/\sqrt{2c}\right) = c\left(\pm 1/\sqrt{2c}\right) e^{-c\left(\pm 1/\sqrt{2c}\right)^2} = \pm\sqrt{c/2e}$. So as c increases, the extreme points become more pronounced. Note that if $c > 0$, then $\lim\limits_{x \to \pm\infty} f(x) = 0$. If $c < 0$, then there are no extreme values, and $\lim\limits_{x \to \pm\infty} f(x) = \mp\infty$.

To find the points of inflection, we differentiate again: $f'(x) = ce^{-cx^2}\left(-2cx^2 + 1\right) \quad \Rightarrow$
$f''(x) = c\left[e^{-cx^2}(-4cx) + (-2cx^2 + 1)(-2cxe^{-cx^2})\right] = -2c^2xe^{-cx^2}(3 - 2cx^2)$. This is 0 at $x = 0$ and where $3 - 2cx^2 = 0 \quad \Leftrightarrow \quad x = \pm\sqrt{3/(2c)} \quad \Rightarrow \quad$ IP at $\left(\pm\sqrt{3/(2c)}, \pm\sqrt{3c/2}\, e^{-3/2}\right)$. If $c > 0$ there are three inflection points, and as c increases, the x-coordinates of the nonzero inflection points approach 0. If $c < 0$, there is only one inflection point, the origin.



87. $s(t) = Ae^{-ct}\cos(\omega t + \delta) \quad \Rightarrow$

$$v(t) = s'(t) = A\{e^{-ct}\left[-\omega\sin(\omega t + \delta)\right] + \cos(\omega t + \delta)(-ce^{-ct})\} = -Ae^{-ct}\left[\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)\right] \quad \Rightarrow$$

$$\begin{aligned} a(t) = v'(t) &= -A\{e^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta)] + [\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)](-ce^{-ct})\} \\ &= -Ae^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c^2\cos(\omega t + \delta)] \\ &= -Ae^{-ct}[(\omega^2 - c^2)\cos(\omega t + \delta) - 2c\omega\sin(\omega t + \delta)] = Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega\sin(\omega t + \delta)] \end{aligned}$$

88.

(a) Let $f(x) = \ln x + x - 3$. Then $f'(x) = 1/x + 1 >$ [for $x > 0$)] and $f(2) \approx -0.307$ and $f(e) \approx 0.718$.

f is differentiable on $(2, e)$, continuous on $[2, e]$ and $f(2) < 0$, $f(e) > 0$. Therefore, by the Intermediate Value Theorem there exists a number c in $(2, e)$ such that $f(c) = 0$. Thus, there is one root. But $f'(x) > 0$ for $x \in (2, e)$, so f is increasing on $(2, e)$, which means that there is exactly one root.

(b) We use Newton's Method with $f(x) = \ln x + x - 3$, $f'(x) = 1/x + 1$, and $x_1 = 2$.

$x_2 = x_1 - \dfrac{\ln x_1 + x_1 - 3}{1/x_1 + 1} = 2 - \dfrac{\ln 2 + 2 - 3}{1/2 + 1} \approx 2.20457$. Similarly, $x_3 \approx 2.20794$, $x_4 = 2.20794$. Thus, the root of the equation, correct to four decimal places, is 2.2079.

89. (a) $y(t) = y(0)e^{kt} = 200e^{kt} \quad \Rightarrow \quad y(0.5) = 200e^{0.5k} = 360 \quad \Rightarrow \quad e^{0.5k} = 1.8 \quad \Rightarrow \quad 0.5k = \ln 1.8 \quad \Rightarrow$
$k = 2\ln 1.8 = \ln(1.8)^2 = \ln 3.24 \quad \Rightarrow \quad y(t) = 200e^{(\ln 3.24)t} = 200(3.24)^t$

(b) $y(4) = 200(3.24)^4 \approx 22{,}040$ bacteria

(c) $y'(t) = 200(3.24)^t \cdot \ln 3.24$, so $y'(4) = 200(3.24)^4 \cdot \ln 3.24 \approx 25{,}910$ bacteria per hour

(d) $200(3.24)^t = 10{,}000 \quad \Rightarrow \quad (3.24)^t = 50 \quad \Rightarrow \quad t\ln 3.24 = \ln 50 \quad \Rightarrow \quad t = \ln 50/\ln 3.24 \approx 3.33$ hours

90. (a) If $y(t)$ is the mass remaining after t years, then $y(t) = y(0)e^{kt} = 100e^{kt}$. $y(5.24) = 100e^{5.24k} = \frac{1}{2} \cdot 100 \quad \Rightarrow$
$e^{5.24k} = \frac{1}{2} \quad \Rightarrow \quad 5.24k = -\ln 2 \quad \Rightarrow \quad k = -\frac{1}{5.24}\ln 2 \quad \Rightarrow \quad y(t) = 100e^{-(\ln 2)t/5.24} = 100 \cdot 2^{-t/5.24}$. Thus, $y(20) = 100 \cdot 2^{-20/5.24} \approx 7.1$ mg.

(b) $100 \cdot 2^{-t/5.24} = 1 \quad \Rightarrow \quad 2^{-t/5.24} = \dfrac{1}{100} \quad \Rightarrow \quad -\dfrac{t}{5.24}\ln 2 = \ln\dfrac{1}{100} \quad \Rightarrow \quad t = 5.24\dfrac{\ln 100}{\ln 2} \approx 34.8$ years

91.

Let $P(t) = \dfrac{64}{1 + 31e^{-0.7944t}} = \dfrac{A}{1 + Be^{ct}} = A(1 + Be^{ct})^{-1}$, where $A = 64$, $B = 31$, and $c = -0.7944$.

$$P'(t) = -A(1 + Be^{ct})^{-2}(Bce^{ct}) = -ABce^{ct}(1 + Be^{ct})^{-2}$$

$$P''(t) = -ABce^{ct}[-2(1 + Be^{ct})^{-3}(Bce^{ct})] + (1 + Be^{ct})^{-2}(-ABc^2e^{ct})$$

$$= -ABc^2e^{ct}(1 + Be^{ct})^{-3}[-2Be^{ct} + (1 + Be^{ct})] = -\frac{ABc^2e^{ct}(1 - Be^{ct})}{(1 + Be^{ct})^3}$$

The population is increasing most rapidly when its graph changes from CU to CD; that is, when $P''(t) = 0$ in this case.

$P''(t) = 0 \quad \Rightarrow \quad Be^{ct} = 1 \quad \Rightarrow \quad e^{ct} = \dfrac{1}{B} \quad \Rightarrow \quad ct = \ln\dfrac{1}{B} \quad \Rightarrow \quad t = \dfrac{\ln(1/B)}{c} = \dfrac{\ln(1/31)}{-0.7944} \approx 4.32$ days. Note that

$P\left(\dfrac{1}{c}\ln\dfrac{1}{B}\right) = \dfrac{A}{1 + Be^{c(1/c)\ln(1/B)}} = \dfrac{A}{1 + Be^{\ln(1/B)}} = \dfrac{A}{1 + B(1/B)} = \dfrac{A}{1 + 1} = \dfrac{A}{2}$, one-half the limit of P as $t \to \infty$.

92. Let $t = 4u$. Then $dt = 4\,du$ and

$$\int_0^4 \frac{1}{16+t^2}\,dt = \int_0^1 \frac{1}{16+16u^2}\cdot 4\,du = \frac{1}{4}\int_0^1 \frac{du}{1+u^2} = \frac{1}{4}\Big[\tan^{-1} u\Big]_0^1 = \tfrac{1}{4}(\tan^{-1} 1 - \tan^{-1} 0) = \tfrac{1}{4}\left(\tfrac{\pi}{4} - 0\right) = \tfrac{\pi}{16}.$$

93. Let $u = -2y^2$. Then $du = -4y\,dy$ and $\int_0^1 ye^{-2y^2}\,dy = \int_0^{-2} e^u\left(-\tfrac{1}{4}\,du\right) = -\tfrac{1}{4}\left[e^u\right]_0^{-2} = -\tfrac{1}{4}(e^{-2}-1) = \tfrac{1}{4}(1-e^{-2})$.

94. $\displaystyle\int_2^5 \frac{dr}{1+2r} = \tfrac{1}{2}\big[\ln|1+2r|\,\big]_2^5 = \tfrac{1}{2}(\ln 11 - \ln 5) = \tfrac{1}{2}\ln\tfrac{11}{5}$

95. Let $u = e^x$, so $du = e^x\,dx$. When $x = 0, u = 1$; when $x = 1, u = e$. Thus,

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \int_1^e \frac{1}{1+u^2}\,du = \big[\arctan u\big]_1^e = \arctan e - \arctan 1 = \arctan e - \tfrac{\pi}{4}.$$

96. Let $u = \sin x$. Then $du = \cos x\,dx$, so

$$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx = \int_0^1 \frac{1}{1+u^2}\,du = \big[\tan^{-1} u\big]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = \tfrac{\pi}{4} - 0 = \tfrac{\pi}{4}.$$

97. Let $u = \sqrt{x}$. Then $du = \dfrac{dx}{2\sqrt{x}} \quad\Rightarrow\quad \displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C.$

98. Let $u = \ln x$. Then $du = \dfrac{dx}{x} \quad\Rightarrow\quad \displaystyle\int \frac{\cos(\ln x)}{x}\,dx = \int \cos u\,du = \sin u + C = \sin(\ln x) + C.$

99. Let $u = x^2 + 2x$. Then $du = (2x+2)\,dx = 2(x+1)\,dx$ and

$$\int \frac{x+1}{x^2+2x}\,dx = \int \frac{\frac{1}{2}\,du}{u} = \tfrac{1}{2}\ln|u| + C = \tfrac{1}{2}\ln\left|x^2+2x\right| + C.$$

100. Let $u = 1 + \cot x$. Then $du = -\csc^2 x\,dx$, so $\displaystyle\int \frac{\csc^2 x}{1+\cot x}\,dx = \int \frac{1}{u}\,(-du) = -\ln|u| + C = -\ln|1+\cot x| + C.$

101. Let $u = \ln(\cos x)$. Then $du = \dfrac{-\sin x}{\cos x}\,dx = -\tan x\,dx \quad\Rightarrow$

$\int \tan x \ln(\cos x)\,dx = -\int u\,du = -\tfrac{1}{2}u^2 + C = -\tfrac{1}{2}[\ln(\cos x)]^2 + C.$

102. Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \tfrac{1}{2}\sin^{-1} u + C = \tfrac{1}{2}\sin^{-1}\left(x^2\right) + C.$

103. Let $u = \tan\theta$. Then $du = \sec^2\theta\,d\theta$ and $\displaystyle\int 2^{\tan\theta}\sec^2\theta\,d\theta = \int 2^u du = \frac{2^u}{\ln 2} + C = \frac{2^{\tan\theta}}{\ln 2} + C.$

104. $\displaystyle\int \sinh au\,du = \frac{1}{a}\cosh au + C$

105. $\int \left(\frac{1-x}{x}\right)^2 dx = \int \left(\frac{1}{x} - 1\right)^2 dx = \int \left(\frac{1}{x^2} - \frac{2}{x} + 1\right) dx = -\frac{1}{x} - 2\ln|x| + x + C$

109. $f(x) = \int_1^{\sqrt{x}} \frac{e^s}{s}\, ds \quad \Rightarrow \quad f'(x) = \frac{d}{dx}\int_1^{\sqrt{x}} \frac{e^s}{s}\, ds = \frac{e^{\sqrt{x}}}{\sqrt{x}} \frac{d}{dx}\sqrt{x} = \frac{e^{\sqrt{x}}}{\sqrt{x}} \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2x}$

110.

$f(x) = \int_{\ln x}^{2x} e^{-t^2}\, dt \quad \Rightarrow$

$f'(x) = \frac{d}{dx}\int_{\ln x}^{2x} e^{-t^2}\, dt = -\frac{d}{dx}\int_0^{\ln x} e^{-t^2}\, dt + \frac{d}{dx}\int_0^{2x} e^{-t^2}\, dt = -e^{-(\ln x)^2}\left(\frac{1}{x}\right) + e^{-(2x)^2}(2) = -\frac{e^{-(\ln x)^2}}{x} + 2e^{-4x^2}$

111. $f_{\text{ave}} = \frac{1}{4-1}\int_1^4 \frac{1}{x}\, dx = \frac{1}{3}\big[\ln|x|\big]_1^4 = \frac{1}{3}[\ln 4 - \ln 1] = \frac{1}{3}\ln 4$

112. $A = \int_{-2}^0 (e^{-x} - e^x)\, dx + \int_0^1 (e^x - e^{-x})\, dx = \left[-e^{-x} - e^x\right]_{-2}^0 + \left[e^x + e^{-x}\right]_0^1$

$= [(-1-1) - (-e^2 - e^{-2})] + [(e + e^{-1}) - (1+1)] = e^2 + e + e^{-1} + e^{-2} - 4$

113. $V = \int_0^1 \frac{2\pi x}{1+x^4}\, dx$ by cylindrical shells. Let $u = x^2 \quad \Rightarrow \quad du = 2x\, dx$. Then

$V = \int_0^1 \frac{\pi}{1+u^2}\, du = \pi\left[\tan^{-1} u\right]_0^1 = \pi\left(\tan^{-1} 1 - \tan^{-1} 0\right) = \pi\left(\frac{\pi}{4}\right) = \frac{\pi^2}{4}.$

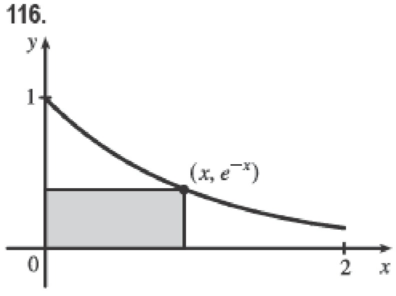
114. $f(x) = x + x^2 + e^x \quad \Rightarrow \quad f'(x) = 1 + 2x + e^x$ and $f(0) = 1 \quad \Rightarrow \quad g(1) = 0$ [where $g = f^{-1}$],

so $g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)} = \frac{1}{2}$.

115. $f(x) = \ln x + \tan^{-1} x \quad \Rightarrow \quad f(1) = \ln 1 + \tan^{-1} 1 = \frac{\pi}{4} \quad \Rightarrow \quad g\left(\frac{\pi}{4}\right) = 1$ [where $g = f^{-1}$].

$f'(x) = \frac{1}{x} + \frac{1}{1+x^2}$, so $g'\left(\frac{\pi}{4}\right) = \frac{1}{f'(1)} = \frac{1}{3/2} = \frac{2}{3}$.

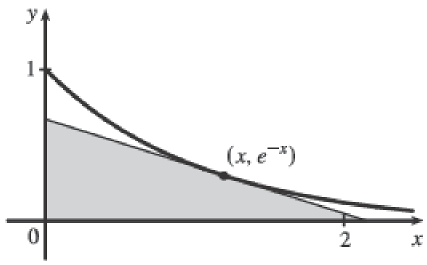
116.



The area of such a rectangle is just the product of its sides, that is, $A(x) = x \cdot e^{-x}$. We want to find the maximum of this function, so we differentiate:

$A'(x) = x(-e^{-x}) + e^{-x}(1) = e^{-x}(1-x)$. This is 0 only at $x = 1$, and changes from positive to negative there, so by the First Derivative Test this gives a local maximum. So the largest area is $A(1) = 1/e$.

117.



We find the equation of a tangent to the curve $y = e^{-x}$, so that we can find the x- and y-intercepts of this tangent, and then we can find the area of the triangle.

The slope of the tangent at the point (a, e^{-a}) is given by $\frac{d}{dx}e^{-x}\Big]_{x=a} = -e^{-a}$,

and so the equation of the tangent is $y - e^{-a} = -e^{-a}(x - a) \quad \Leftrightarrow$

$y = e^{-a}(a - x + 1)$.

The y-intercept of this line is $y = e^{-a}(a - 0 + 1) = e^{-a}(a + 1)$. To find the x-intercept we set $y = 0 \quad \Rightarrow$ $e^{-a}(a - x + 1) = 0 \quad \Rightarrow \quad x = a + 1$. So the area of the triangle is $A(a) = \frac{1}{2}\left[e^{-a}(a + 1)\right](a + 1) = \frac{1}{2}e^{-a}(a + 1)^2$. We differentiate this with respect to a: $A'(a) = \frac{1}{2}\left[e^{-a}(2)(a + 1) + (a + 1)^2 e^{-a}(-1)\right] = \frac{1}{2}e^{-a}\left(1 - a^2\right)$. This is 0 at $a = \pm 1$, and the root $a = 1$ gives a maximum, by the First Derivative Test. So the maximum area of the triangle is $A(1) = \frac{1}{2}e^{-1}(1 + 1)^2 = 2e^{-1} = 2/e$.