21. $f(t) = t^2 \ln t \quad \Rightarrow \quad f'(t) = t^2 \cdot \dfrac{1}{t} + (\ln t)(2t) = t + 2t\ln t$ or $t(1 + 2\ln t)$

22. $g(t) = \dfrac{e^t}{1+e^t} \quad \Rightarrow \quad g'(t) = \dfrac{(1+e^t)e^t - e^t(e^t)}{(1+e^t)^2} = \dfrac{e^t}{(1+e^t)^2}$

23. $h(\theta) = e^{\tan 2\theta} \quad \Rightarrow \quad h'(\theta) = e^{\tan 2\theta} \cdot \sec^2 2\theta \cdot 2 = 2\sec^2(2\theta)\, e^{\tan 2\theta}$

24. $h(u) = 10^{\sqrt{u}} \quad \Rightarrow \quad h'(u) = 10^{\sqrt{u}} \cdot \ln 10 \cdot \dfrac{1}{2\sqrt{u}} = \dfrac{(\ln 10)10^{\sqrt{u}}}{2\sqrt{u}}$

25. $y = \ln|\sec 5x + \tan 5x| \quad \Rightarrow$

$$y' = \frac{1}{\sec 5x + \tan 5x}(\sec 5x\ \tan 5x \cdot 5 + \sec^2 5x \cdot 5) = \frac{5\sec 5x\,(\tan 5x + \sec 5x)}{\sec 5x + \tan 5x} = 5\sec 5x$$

26. $y = x\cos^{-1} x \quad \Rightarrow \quad y' = x\left(-\dfrac{1}{\sqrt{1-x^2}}\right) + (\cos^{-1} x)(1) = \cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}}$

27. $y = x\tan^{-1}(4x) \quad \Rightarrow \quad y' = x \cdot \dfrac{1}{1+(4x)^2} \cdot 4 + \tan^{-1}(4x) \cdot 1 = \dfrac{4x}{1+16x^2} + \tan^{-1}(4x)$

28. $y = e^{mx}\cos nx \quad \Rightarrow$

$$y' = e^{mx}(\cos nx)' + \cos nx\,(e^{mx})' = e^{mx}(-\sin nx \cdot n) + \cos nx\,(e^{mx} \cdot m) = e^{mx}(m\cos nx - n\sin nx)$$

29. $y = \ln(\sec^2 x) = 2\ln|\sec x| \quad \Rightarrow \quad y' = (2/\sec x)(\sec x \tan x) = 2\tan x$

30. $y = \sqrt{t\ln(t^4)} \quad \Rightarrow$

$$y' = \frac{1}{2}[t\ln(t^4)]^{-1/2}\,\frac{d}{dt}\,[t\ln(t^4)] = \frac{1}{2\sqrt{t\ln(t^4)}} \cdot \left[1 \cdot \ln(t^4) + t \cdot \frac{1}{t^4} \cdot 4t^3\right] = \frac{1}{2\sqrt{t\ln(t^4)}} \cdot [\ln(t^4) + 4] = \frac{\ln(t^4)+4}{2\sqrt{t\ln(t^4)}}$$

Or: Since y is only defined for $t > 0$, we can write $y = \sqrt{t \cdot 4\ln t} = 2\sqrt{t\ln t}$. Then

$y' = 2 \cdot \dfrac{1}{2\sqrt{t\ln t}} \cdot \left(1 \cdot \ln t + t \cdot \dfrac{1}{t}\right) = \dfrac{\ln t + 1}{\sqrt{t\ln t}}$. This agrees with our first answer since

$$\frac{\ln(t^4)+4}{2\sqrt{t\ln(t^4)}} = \frac{4\ln t + 4}{2\sqrt{t \cdot 4\ln t}} = \frac{4(\ln t + 1)}{2 \cdot 2\sqrt{t\ln t}} = \frac{\ln t + 1}{\sqrt{t\ln t}}.$$

31. $y = \dfrac{e^{1/x}}{x^2} \quad \Rightarrow \quad y' = \dfrac{x^2(e^{1/x})' - e^{1/x}\,(x^2)'}{(x^2)^2} = \dfrac{x^2(e^{1/x})(-1/x^2) - e^{1/x}(2x)}{x^4} = \dfrac{-e^{1/x}(1+2x)}{x^4}$

32. $y = (\arcsin 2x)^2 \quad \Rightarrow \quad y' = 2(\arcsin 2x) \cdot (\arcsin 2x)' = 2\arcsin 2x \cdot \dfrac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \dfrac{4\arcsin 2x}{\sqrt{1-4x^2}}$

33. $y = 3^{x\ln x} \quad\Rightarrow\quad y' = 3^{x\ln x}(\ln 3)\,\dfrac{d}{dx}(x\ln x) = 3^{x\ln x}(\ln 3)\left(x\cdot\dfrac{1}{x} + \ln x\cdot 1\right) = 3^{x\ln x}(\ln 3)(1+\ln x)$

34. $y = e^{\cos x} + \cos(e^x) \quad\Rightarrow\quad y' = -\sin x\, e^{\cos x} - e^x\sin(e^x)$

35. $H(v) = v\tan^{-1} v \quad\Rightarrow\quad H'(v) = v\cdot\dfrac{1}{1+v^2} + \tan^{-1} v\cdot 1 = \dfrac{v}{1+v^2} + \tan^{-1} v$

36. $F(z) = \log_{10}(1+z^2) \quad\Rightarrow\quad F'(z) = \dfrac{1}{(\ln 10)(1+z^2)}\cdot 2z = \dfrac{2z}{(\ln 10)(1+z^2)}$

37. $y = x\sinh(x^2) \quad\Rightarrow\quad y' = x\cosh(x^2)\cdot 2x + \sinh(x^2)\cdot 1 = 2x^2\cosh(x^2) + \sinh(x^2)$

38. $y = (\cos x)^x \quad\Rightarrow\quad \ln y = \ln(\cos x)^x = x\ln\cos x \quad\Rightarrow\quad \dfrac{y'}{y} = x\cdot\dfrac{1}{\cos x}\cdot(-\sin x) + \ln\cos x\cdot 1 \quad\Rightarrow$

$y' = (\cos x)^x(\ln\cos x - x\tan x)$

39. $y = \ln\sin x - \frac{1}{2}\sin^2 x \quad\Rightarrow\quad y' = \dfrac{1}{\sin x}\cdot\cos x - \frac{1}{2}\cdot 2\sin x\cdot\cos x = \cot x - \sin x\,\cos x$

40. $y = \arctan\left(\arcsin\sqrt{x}\right) \quad\Rightarrow\quad y' = \dfrac{1}{1+\left(\arcsin\sqrt{x}\right)^2}\cdot\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{x}}$

41.

$y = \ln\left(\dfrac{1}{x}\right) + \dfrac{1}{\ln x} = \ln x^{-1} + (\ln x)^{-1} = -\ln x + (\ln x)^{-1} \quad\Rightarrow\quad y' = -1\cdot\dfrac{1}{x} + (-1)(\ln x)^{-2}\cdot\dfrac{1}{x} = -\dfrac{1}{x} - \dfrac{1}{x(\ln x)^2}$

42. $xe^y = y - 1 \quad\Rightarrow\quad e^y + xe^y y' = y' \quad\Rightarrow\quad y' = e^y/(1-xe^y)$

43. $y = \ln(\cosh 3x) \quad\Rightarrow\quad y' = (1/\cosh 3x)(\sinh 3x)(3) = 3\tanh 3x$

44. $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} \quad\Rightarrow$

$$\ln y = \ln\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5} = \ln(x^2+1)^4 - \ln[(2x+1)^3(3x-1)^5] = 4\ln(x^2+1) - [\ln(2x+1)^3 + \ln(3x-1)^5]$$

$$= 4\ln(x^2+1) - 3\ln(2x+1) - 5\ln(3x-1) \quad\Rightarrow$$

$$\frac{y'}{y} = 4\cdot\frac{1}{x^2+1}\cdot 2x - 3\cdot\frac{1}{2x+1}\cdot 2 - 5\cdot\frac{1}{3x-1}\cdot 3 \quad\Rightarrow\quad y' = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}\left(\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right).$$

[The answer could be simplified to $y' = -\dfrac{(x^2+56x+9)(x^2+1)^3}{(2x+1)^4(3x-1)^6}$, but this is unnecessary.]

45. $y = \cosh^{-1}(\sinh x) \quad \Rightarrow \quad y' = (\cosh x)/\sqrt{\sinh^2 x - 1}$

46. $y = x \tanh^{-1}\sqrt{x} \quad \Rightarrow \quad y' = \tanh^{-1}\sqrt{x} + x\dfrac{1}{1-(\sqrt{x})^2}\dfrac{1}{2\sqrt{x}} = \tanh^{-1}\sqrt{x} + \dfrac{\sqrt{x}}{2(1-x)}$

47. $y = \cos\left(e^{\sqrt{\tan 3x}}\right) \quad \Rightarrow$

$$y' = -\sin\left(e^{\sqrt{\tan 3x}}\right)\cdot\left(e^{\sqrt{\tan 3x}}\right)' = -\sin\left(e^{\sqrt{\tan 3x}}\right)\, e^{\sqrt{\tan 3x}}\cdot\tfrac{1}{2}(\tan 3x)^{-1/2}\cdot\sec^2(3x)\cdot 3$$
$$= \frac{-3\sin\left(e^{\sqrt{\tan 3x}}\right) e^{\sqrt{\tan 3x}}\ \sec^2(3x)}{2\sqrt{\tan 3x}}$$

48. $\dfrac{d}{dx}\left(\dfrac{1}{2}\tan^{-1}x + \dfrac{1}{4}\ln\dfrac{(x+1)^2}{x^2+1}\right) = \dfrac{d}{dx}\left(\dfrac{1}{2}\tan^{-1}x + \dfrac{1}{2}\ln|x+1| - \dfrac{1}{4}\ln(x^2+1)\right)$

$$= \frac{1}{2}\frac{1}{x^2+1} + \frac{1}{2}\frac{1}{x+1} - \frac{1}{4}\frac{2x}{x^2+1} = \frac{1}{2}\left(\frac{1}{x^2+1} - \frac{x}{x^2+1} + \frac{1}{x+1}\right)$$
$$= \frac{1}{2}\left(\frac{1-x}{x^2+1} + \frac{1}{x+1}\right) = \frac{1}{2}\left(\frac{1-x^2}{(x^2+1)(1+x)} + \frac{x^2+1}{(x^2+1)(1+x)}\right)$$
$$= \frac{1}{2}\frac{2}{(x^2+1)(1+x)} = \frac{1}{(1+x)(x^2+1)}$$

49. $f(x) = e^{g(x)} \quad \Rightarrow \quad f'(x) = e^{g(x)}g'(x)$

50. $f(x) = g(e^x) \quad \Rightarrow \quad f'(x) = g'(e^x)\,e^x$

51. $f(x) = \ln|g(x)| \quad \Rightarrow \quad f'(x) = \dfrac{1}{g(x)}g'(x) = \dfrac{g'(x)}{g(x)}$

52. $f(x) = g(\ln x) \quad \Rightarrow \quad f'(x) = g'(\ln x)\cdot\dfrac{1}{x} = \dfrac{g'(\ln x)}{x}$

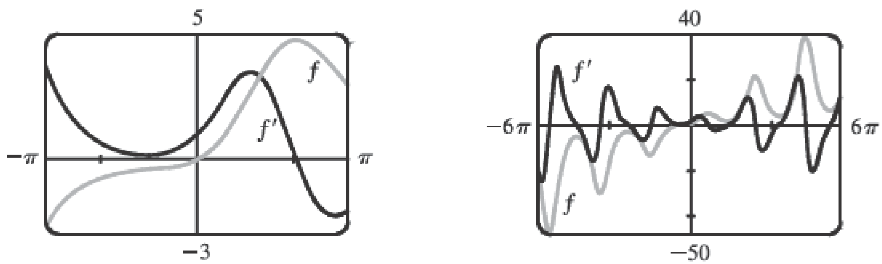
57.
$y = (2+x)e^{-x} \quad \Rightarrow \quad y' = (2+x)(-e^{-x}) + e^{-x}\cdot 1 = e^{-x}[-(2+x)+1] = e^{-x}(-x-1)$. At $(0,2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x-0)$, or $y = -x+2$.

58. $y = f(x) = x\ln x \quad \Rightarrow \quad f'(x) = \ln x + 1$, so the slope of the tangent at (e,e) is $f'(e) = 2$ and an equation is $y - e = 2(x-e)$ or $y = 2x - e$.

59. $y = [\ln(x+4)]^2 \quad \Rightarrow \quad y' = 2[\ln(x+4)]^1\cdot\dfrac{1}{x+4}\cdot 1 = 2\,\dfrac{\ln(x+4)}{x+4}$ and $y' = 0 \quad \Leftrightarrow \quad \ln(x+4) = 0 \quad \Leftrightarrow$

$x + 4 = e^0 \quad \Rightarrow \quad x+4 = 1 \quad \Leftrightarrow \quad x = -3$, so the tangent is horizontal at the point $(-3, 0)$.

60.
$f(x) = xe^{\sin x} \quad \Rightarrow \quad f'(x) = x[e^{\sin x}(\cos x)] + e^{\sin x}(1) = e^{\sin x}(x\cos x + 1)$. As a check on our work, we notice from the graphs that $f'(x) > 0$ when f is increasing. Also, we see in the larger viewing rectangle a certain similarity in the graphs of f and f': the sizes of the oscillations of f and f' are linked.



61.
(a) The line $x - 4y = 1$ has slope $\frac{1}{4}$. A tangent to $y = e^x$ has slope $\frac{1}{4}$ when $y' = e^x = \frac{1}{4} \quad \Rightarrow \quad x = \ln\frac{1}{4} = -\ln 4$.
Since $y = e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4, \frac{1}{4}\right)$. Thus, an equation of the tangent line is $y - \frac{1}{4} = \frac{1}{4}(x + \ln 4)$ or $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$.

(b) The slope of the tangent at the point (a, e^a) is $\left.\frac{d}{dx}e^x\right|_{x=a} = e^a$. Thus, an equation of the tangent line is
$y - e^a = e^a(x - a)$. We substitute $x = 0, y = 0$ into this equation, since we want the line to pass through the origin:
$0 - e^a = e^a(0 - a) \quad \Leftrightarrow \quad -e^a = e^a(-a) \quad \Leftrightarrow \quad a = 1$. So an equation of the tangent line at the point $(a, e^a) = (1, e)$ is $y - e = e(x - 1)$ or $y = ex$.

62. (a) $\lim_{t\to\infty} C(t) = \lim_{t\to\infty}[K(e^{-at} - e^{-bt})] = K\lim_{t\to\infty}(e^{-at} - e^{-bt}) = K(0 - 0) = 0$ because $-at \to -\infty$ and $-bt \to -\infty$ as $t \to \infty$.

(b) $C(t) = K(e^{-at} - e^{-bt}) \quad \Rightarrow \quad C'(t) = K(e^{-at}(-a) - e^{-bt}(-b)) = K(-ae^{-at} + be^{-bt})$

(c) $C'(t) = 0 \quad \Leftrightarrow \quad be^{-bt} = ae^{-at} \quad \Leftrightarrow \quad \frac{b}{a} = e^{(-a+b)t} \quad \Leftrightarrow \quad \ln\frac{b}{a} = (b - a)t \quad \Leftrightarrow \quad t = \frac{\ln(b/a)}{b - a}$

63. $\lim_{x\to\infty} e^{-3x} = 0$ since $-3x \to -\infty$ as $x \to \infty$ and $\lim_{t\to-\infty} e^t = 0$.

64. $\lim_{x\to 10^-} \ln(100 - x^2) = -\infty$ since as $x \to 10^-$, $(100 - x^2) \to 0^+$.

65. Let $t = 2/(x - 3)$. As $x \to 3^-$, $t \to -\infty$. $\lim_{x\to 3^-} e^{2/(x-3)} = \lim_{t\to-\infty} e^t = 0$

66. If $y = x^3 - x = x(x^2 - 1)$, then as $x \to \infty$, $y \to \infty$. $\lim_{x\to\infty}\arctan(x^3 - x) = \lim_{y\to\infty}\arctan y = \frac{\pi}{2}$.

67. Let $t = \sinh x$. As $x \to 0^+$, $t \to 0^+$. $\lim\limits_{x\to 0^+} \ln(\sinh x) = \lim\limits_{t\to 0^+} \ln t = -\infty$

68. $-1 \le \sin x \le 1 \quad \Rightarrow \quad -e^{-x} \le e^{-x}\sin x \le e^{-x}$. Now $\lim\limits_{x\to\infty} \left(\pm e^{-x}\right) = 0$, so by the Squeeze Theorem,

$\lim\limits_{x\to\infty} e^{-x}\sin x = 0$.

69. $\lim\limits_{x\to\infty} \dfrac{(1+2^x)/2^x}{(1-2^x)/2^x} = \lim\limits_{x\to\infty} \dfrac{1/2^x + 1}{1/2^x - 1} = \dfrac{0+1}{0-1} = -1$

70. Let $t = x/4$, so $x = 4t$. As $x \to \infty$, $t \to \infty$. $\lim\limits_{x\to\infty} \left(1 + \dfrac{4}{x}\right)^x = \lim\limits_{t\to\infty} \left(1 + \dfrac{1}{t}\right)^{4t} = \left[\lim\limits_{t\to\infty} \left(1 + \dfrac{1}{t}\right)^t\right]^4 = e^4$

71. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{e^x - 1}{\tan x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{e^x}{\sec^2 x} = \dfrac{1}{1} = 1$

72. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{1 - \cos x}{x^2 + x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{2x + 1} = \dfrac{0}{1} = 0$

73. This limit has the form $\frac{0}{0}$. $\lim\limits_{x\to 0} \dfrac{e^{4x} - 1 - 4x}{x^2} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{4e^{4x} - 4}{2x} \overset{\text{H}}{=} \lim\limits_{x\to 0} \dfrac{16e^{4x}}{2} = \lim\limits_{x\to 0} 8e^{4x} = 8 \cdot 1 = 8$

74. This limit has the form $\frac{\infty}{\infty}$. $\lim\limits_{x\to\infty} \dfrac{e^{4x} - 1 - 4x}{x^2} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{4e^{4x} - 4}{2x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{16e^{4x}}{2} = \lim\limits_{x\to\infty} 8e^{4x} = \infty$