45. Simplify each:

$\sin\theta(\csc\theta - \sin\theta)$

$= \sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$

$= 1 - \sin^2\theta$

$= \boxed{\cos^2\theta}$

$\tan\theta\cos^2\theta\sec\theta.$

$= \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta}{1}\right)\left(\frac{1}{\cos\theta}\right)$

$= \boxed{\sin\theta}$

$\frac{\cos\theta}{\sec\theta} + \frac{\sin\theta}{\csc\theta}$

$= \frac{\cos\theta}{\frac{1}{\cos\theta}} + \frac{\sin\theta}{\frac{1}{\sin\theta}}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

$\frac{\sec^2 x}{\cot^2 x + 1} = \frac{\sec^2 x}{\csc^2 x}$

$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} = \frac{\sin^2 x}{\cos^2 x}$

$= \boxed{\tan^2 x}$

$\frac{(1+\sin x)}{(1+\sin x)}\frac{1}{1-\sin x} + \frac{1}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)}$

$\frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x} = \frac{2}{1-\sin^2 x}$

$= \frac{2}{\cos^2 x} = \boxed{\sec^2 x}$

$\cot^2 x\csc^2 x - \cot^2 x$

$\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\cos^2 x}{\sin^4 x} - \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\cos^2 x}{\sin^4 x} - \frac{\cos^2 x\sin^2 x}{\sin^4 x} = \frac{\cos^2 x - \cos^2 x\sin^2 x}{\sin^4 x}$

$= \frac{\cos^2 x(1-\sin^2 x)}{\sin^4 x} = \frac{\cos^2 x(\cos^2 x)}{\sin^4 x} = \frac{\cos^4 x}{\sin^4 x}$

$= \boxed{\cot^4 x}$

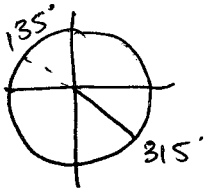
46. Solve for ALL solutions in radians

$\tan\theta+1=0$

$\tan\theta = -1$

$\tan^{-1}(-1) = \theta$

$\theta = -45°$



$\theta = 135° + 180K$

in Radians

$$\theta = \frac{3\pi}{4} + \pi K$$

$2\cos^2 x - \sin x - 1 = 0$

$2(1-\sin^2 x) - \sin x - 1 = 0$

$2 - 2\sin^2 x - \sin x - 1 = 0$

$-2\sin^2 x - \sin x + 1 = 0$

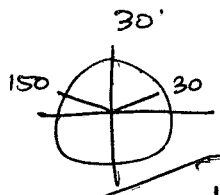
$2\sin^2 x + \sin x - 1 = 0$

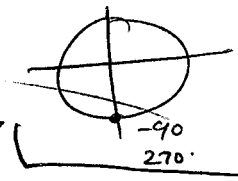
$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ | $\sin x + 1 =$

$\sin x = 1/2$ | $\sin x = -1$

$x = -90$





IN RADIANS

$\pi/6 + 2\pi K$, $5\pi/6 + 2\pi K$ $3\pi/2 + 2\pi K$

$7\cot x - \sqrt{3} = 4\cot x$

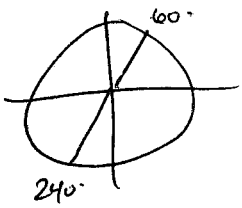
$3\cot x = \sqrt{3}$

$\cot x = \frac{\sqrt{3}}{3}$

$\tan x = \frac{3}{\sqrt{3}}$

$\tan^{-1}(3/\sqrt{3}) = x$

$x = 60$



$$x = \pi/3 + \pi K$$

$9 + \cot^2 x = 12$

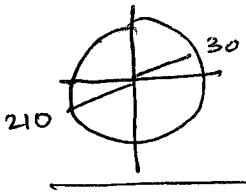
$\cot^2 x = 3$

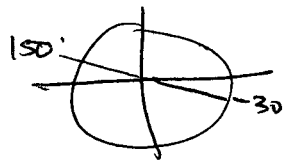
$\cot x = \pm\sqrt{3}$

$\cot x = \sqrt{3}$ | $\cot x = -\sqrt{3}$

$\tan x = \frac{1}{\sqrt{3}}$ | $\tan x = -\frac{1}{\sqrt{3}}$

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$ | $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -30$





IN RADIANS

$\pi/6 + \pi K$ OR $\frac{5\pi}{6} + \pi K$

49. Solve for $0° \le \theta < 360°$.

$2\cos^2\theta + 3\cos\theta + 1 = 0$

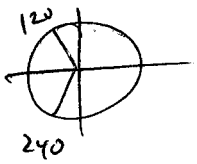
$(2\cos\theta + 1)(\cos\theta + 1) = 0$

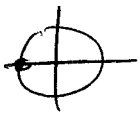
$2\cos\theta + 1 = 0$ $\quad$ $\cos\theta + 1 = 0$

$\cos\theta = -1/2$ $\quad$ $\cos\theta = -1$

$\cos^{-1}(-1/2) = \theta$ $\quad$ $\cos^{-1}(-1) = \theta$

$\theta = 120°$ $\quad$ $\theta = 180°$





$120°, 180°, 240°$

$\sin^2\theta + \cos\theta - 1 = 0$

$1 - \cos^2\theta + \cos\theta - 1 = 0$

$-\cos^2\theta + \cos\theta = 0$

$\cos\theta(-\cos\theta + 1) = 0$

$\cos\theta = 0$ $\quad$ $-\cos\theta + 1 = 0$

$\cos^{-1}(0) = \theta$ $\quad$ $\cos\theta = 1$

$\theta = 90°, 270°$ $\quad$ $\cos^{-1}(1) = \theta$

$\theta = 0°$

$0°, 90°, 270°$

50. Solve for $0 \le x < 2\pi$

$1 = \cot^2 x + \csc x$

$1 = \csc^2 x - 1 + \csc x$

$0 = \csc^2 x + \csc x - 2$

$(\csc x + 2)(\csc x - 1)$

$\csc x + 2 = 0$ $\quad$ $\csc x - 1 = 0$

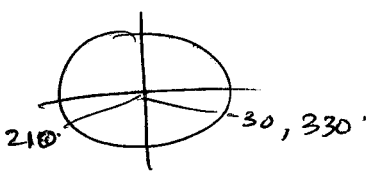
$\csc x = -2$ $\quad$ $\csc x = 1$

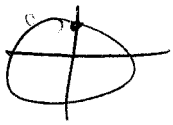
$\frac{1}{\sin x} = -2$ $\quad$ $\frac{1}{\sin x} = 1$

$\sin x = -1/2$ $\quad$ $\sin x = 1$

$\sin^{-1}(-1/2) = x$ $\quad$ $\sin^{-1}(1) = x$

$x = -30$ $\quad$ $x = 90°$





IN RADIANS

$\pi/6, 11\pi/6, \pi/2$

$\frac{3\sin x}{3} = \frac{3 - 3\cos x}{3}$

$\sin x = 1 - \cos x$ $\quad$ SQUARE BOTH SIDES

$\sin^2 x = 1 - 2\cos x + \cos^2 x$

$1 - \cos^2 x = 1 - 2\cos x + \cos^2 x$

$0 = -2\cos x + 2\cos^2 x$

$0 = -2\cos x(1 - \cos x)$

$-2\cos x = 0$ $\quad$ $1 - \cos x = 0$

$\cos x = 0$ $\quad$ $\cos x = 1$

$\cos^{-1}(0) = x$ $\quad$ $\cos^{-1}(1) = x$

$90, 270°$ $\quad$ $0°$

IN RADIANS

$\pi/2, 3\pi/2, 0$

CHECK: $0, \pi/2$ only

$3\pi/2$ IS EXTRANEOUS

47. Solve for $0° \le \theta < 360°$.

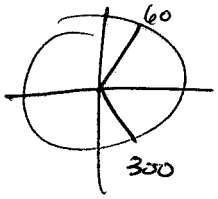
$2\cos\theta - 1 = 0$

$2\cos\theta = 1$

$\cos\theta = 1/2$

$\cos^{-1}(1/2) = \theta$

$\theta = 60$



$60°, 300°$

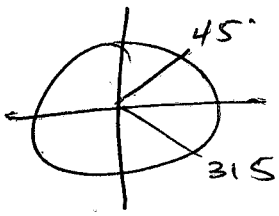
$5\cos x - \sqrt{2} = 3\cos x$

$2\cos x = \sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$x = 45°$



$45°, 315°$

48. Solve for $0 \le \theta < 2\pi$.

$\sin\theta\tan\theta = \sin\theta$

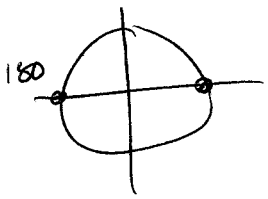
$\sin\theta\tan\theta - \sin\theta = 0$

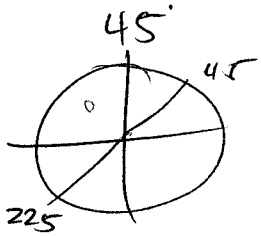
$\sin\theta(\tan\theta - 1) = 0$

$\sin\theta = 0$ $\qquad$ $\tan\theta - 1 = 0$

$\sin^{-1}(0) = \theta$ $\qquad$ $\tan\theta = 1$

0 $\qquad$ $\tan^{-1}(1) = \theta$





In radians

$0, \pi, \pi/4, 5\pi/4$

$4\cot x = \cot x \sin^2 x$

$4\cot x - \cot x \sin^2 x = 0$

$\cot x(4 - \sin^2 x) = 0$

$\cot x = 0$ $\qquad$ $4 - \sin^2 x = 0$

$\frac{1}{\tan x} = \frac{0}{1}$ $\qquad$ $\sin^2 x = 4$

$\tan x = \frac{1}{0}$ $\qquad$ $\sin x = \pm 2$ (NOT POSSIBLE)

$\tan x$ is undefined

$x = 90°, 270°$

$x = \pi/2, 3\pi/2$