Pre Calculus First Semester Midterm Review Name_____

1. What is the domain of

$f(x) = \sqrt{6-x}$

D: $6-x \geq 0$

$-x \geq -6$

$\boxed{x \leq 6}$

$g(x) = \dfrac{3x-1}{x^2-9} = \dfrac{3x-1}{(x+3)(x-3)}$

$\boxed{D: x \neq \pm 3}$

2. Given the functions $f(x) = 3x + 1$ and $g(x) = x^2 + 4$, determine

$f(g(2))$

$= f(2^2+4)$

$= f(8)$

$= \boxed{25}$

$(f+g)(x)$

$= f(x) + g(x)$

$3x+1 + x^2+4$

$= \boxed{x^2+3x+5}$

$g(f(x))$

$= g(3x+1)$

$= \boxed{(3x+1)^2+4}$ OR

$\boxed{9x^2+6x+5}$

3. Given the functions $f(x) = 2x^3$, $g(x) = 5x^2 + 3$ and $h(x) = \dfrac{1}{(x-4)^2} + 3$, determine if the functions are **odd, even or neither.**

$f(x)$

$-f(x) = 2(-x)^3$

$-f(x) = -2x^3$

$f(x) = 2x^3$ ODD

$g(x)$

$g(x) = 5(-x)^2+3$

$g(x) = 5x^2+3$

EVEN

$h(x)$

$h(x) = \dfrac{1}{(-x-4)^2} + 3$

$= \dfrac{1}{(x+4)^2} + 3$

NOT SAME

NEITHER

4. Determine whether the graph of $x^2y = 4$ is symmetric in the ***x-axis***, ***y-axis***, ***origin***, and/or the line $y = x$.

x-axis

$y \to -y$

$x^2(-y) = 4$

$-x^2y = 4$

NO

y-axis

$x \to -x$

$(-x)^2y = 4$

$x^2y = 4$

YES

origin

$x \to -x$

$y \to -y$

$(-x)^2(-y) = 4$

$-x^2y = 4$

NO

$y = x$

x SWITCHES WITH y

$y^2x = 4$

NO

5. Describe the transformations of each:

$p(x) = -3|x+2| - 1$

LEFT 2

DOWN 1

VERT. STRETCH OF 3

VERT REFLECTION

$t(x) = 3\sin\left(2(x-40°)\right) - 6$

RIGHT 40°

DOWN 6

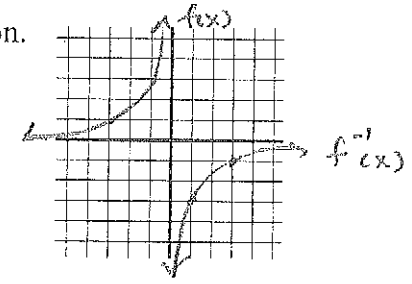
HOR. STRETCH of 1/2

VERT. STRETCH of 3

6. Draw a graph that has an inverse that is also a function.

MUST DRAW A

1 TO 1 FUNCTION



1 TO 1 FUNCTIONS PASS

BOTH VERTICAL AND HORIZONTAL LINE TESTS

7. Describe the discontinuities of each of the following functions. Be sure to include where they occur and their type.

$f(x) = \frac{x-2}{x^2+x-6}$

$= \frac{x-2}{(x-2)(x+3)}$

Removable (Hole) at $x = 2$

Infinite (Asymptote) at $x = -3$

$g(x) = \frac{x-1}{x^2+4x-5}$

$\frac{x-1}{(x-1)(x+5)}$

Removable at $x = 1$

Infinite at $x = -5$

$h(x) = \begin{cases} 2x-1 & x < 3 \\ \frac{2}{x+1} & x \geq 3 \end{cases}$

$= 5$ at $x = 3$

$= \frac{1}{2}$ at $x = 3$

$x = -1$ not in domain

Jump at $x = 3$

8. Given $g(x) = \begin{cases} 2x-1 \text{ if } x < 0 \\ x^2 \text{ if } 0 \leq x \leq 5 \\ \sqrt{x} \text{ if } x > 5 \end{cases}$ find the following:

$= -1$ at $x = 0$

$= 0$ at $x = 0$ and $= 25$ at $x = 5$

$= \sqrt{5}$ at $x = 5$

$g(3)$

$3^2 = \boxed{9}$

$g(0)$

$0^2 = \boxed{0}$

is $g(x)$ continuous?

Check for jumps

No jump at $x = 0$

Jump at $x = 5$

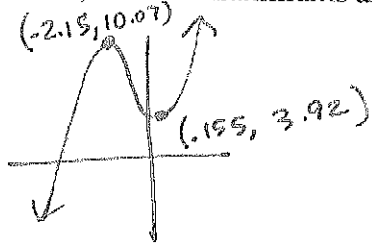
9. Describe the end behavior of the graph of $f(x) = 2x^4 - 2x^2 + x + 4$.

As $x \to \infty$, $f(x) \to +\infty$

As $x \to -\infty$, $f(x) \to +\infty$

10. Using your calculator, find the maximums and minimums of $f(x) = x^3 + 3x^2 - x + 4$? Are they relative or absolute?

Graph:



Relative max at 10.07

Relative min at 3.92

11. The height of a ball thrown straight up into the air can be modeled by the function $h(t) = -4.9t^2 + 12t + 1$, where t is the time in seconds after release and $h(t)$ is the height of the ball in meters. Find the following:

a. average rate of change in meters per second from 2 to 2.5 seconds.

$h(2) = 5.4$ $\quad (2, 5.4)$

$h(2.5) = .375$ $\quad (2.5, .375)$

$\frac{5.4 - .375}{2 - 2.5} = \frac{5.025}{-.5} = \boxed{-10.05}$

b. When does the ball hit the ground?

Set $H = 0$

$0 = -4.9t^2 + 12t + 1$ use Quad. Form. to solve

$\frac{-12 \pm \sqrt{12^2 - 4(-4.9)(1)}}{2(-4.9)} = \frac{-12 \pm \sqrt{163.6}}{-9.8} = \frac{-12 \pm 12.79}{-9.8} =$

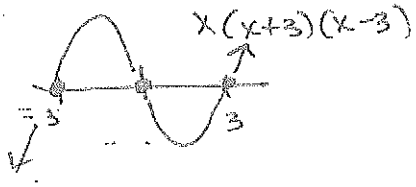
$\boxed{2.53 \text{ sec}}$

12. Solve each: $x^3 - 9x \le 0$

GRAPH

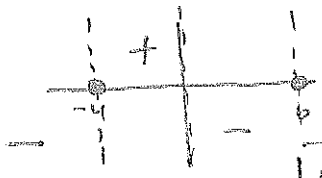
$x(x^2-9)$

$x(x+3)(x-3)$



$(-\infty, -3]$ or $[0, 3]$

$\frac{x^2-2x-24}{x} > 0$ $\qquad \frac{(x+4)(x-6)}{x}$



$(-4, 0)$ or $(6, \infty)$

13. If $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$ has a zero of -1, with a multiplicity of 3, what is another zero of f?

work 3 times

$$\begin{array}{r|ccccc} -1 & 1 & 1 & -3 & -5 & -2 \\ & & -1 & 0 & 3 & 2 \\ \hline -1 & 1 & 0 & -3 & -2 & 0 \\ & & -1 & 1 & 2 & \\ \hline & 1 & -1 & -2 & 0 & \end{array}$$

$$\begin{array}{r|ccc} -1 & 1 & -1 & -2 \\ & & -1 & 2 \\ \hline & 1 & -2 & 0 \end{array}$$

$x - 2 = 0$

$x = 2$

14. Describe the domain of:

$f(x) = \frac{1}{3}x^{-5}$

$= \frac{1}{3x^5}$ $\quad x \neq 0$

$g(x) = \frac{1}{x\sqrt{x-4}}$

$x \neq 0$ $\qquad x - 4 \geq 0$

$x \geq 4$

$x > 4, \; x \neq 0$

15. Divide $(x^4 + x^3 - 3x^2 - 5x - 2)$ by $(x + 1)$.

$$\begin{array}{r|ccccc} -1 & 1 & 1 & -3 & -5 & -2 \\ & & -1 & 0 & 3 & 2 \\ \hline & 1 & 0 & -3 & -2 & 0 \end{array}$$

$\frac{x^4 + x^3 - 3x^2 - 5x - 2}{x + 1} = x^3 - 3x - 2$

16. State the number of possible real zeros and the maximum number of turning points of $g(x) = x^4 - 6x^2 - 16x$.

4 zeros, 0 imaginary

or

2 zeros, 2 imaginary

or 0 zeros, 4 imaginary

17. If $p(x) = 3x^4 + 5x^3 + ax^2 - 80x + 32$ is divided by $(x + 2)$ there is no remainder. Find the value of a

$$\begin{array}{r|ccccc} -2 & 3 & 5 & a & -80 & 32 \\ & & -6 & 2 & -2a-4 & 4a+168 \\ \hline & 3 & -1 & a+2 & -2a-84 & 0 \end{array}$$

$32 + 4a + 168 = 0$

$4a + 200 = 0$

$4a = -200$

$a = -50$

18. Is $(x + 3)$ a factor of $(x) = x^4 + x^3 - 3x^2 - 5x - 2$?

$$\begin{array}{r|ccccc} -3 & 1 & 1 & -3 & -5 & -2 \\ & & -3 & 6 & -9 & 42 \\ \hline & 1 & -2 & 3 & -14 & 40 \end{array}$$

$\leftarrow$ no