1. $\displaystyle\int_1^2 \frac{(x+1)^2}{x}\,dx = \int_1^2 \frac{x^2+2x+1}{x}\,dx = \int_1^2 \left(x+2+\frac{1}{x}\right)dx = \left[\frac{1}{2}x^2+2x+\ln|x|\right]_1^2$

$= (2+4+\ln 2) - \left(\frac{1}{2}+2+0\right) = \frac{7}{2}+\ln 2$

3. $\int_0^{\pi/2} \sin\theta\, e^{\cos\theta}\,d\theta = \int_1^0 e^u\,(-du)$ $\quad\left[\begin{array}{l} u = \cos\theta, \\ du = -\sin\theta\,d\theta \end{array}\right]$

$= \int_0^1 e^u\,du = \left[e^u\right]_0^1 = e^1 - e^0 = e-1$

5. $\displaystyle\int \frac{dt}{2t^2+3t+1} = \int \frac{1}{(2t+1)(t+1)}\,dt = \int \left(\frac{2}{2t+1} - \frac{1}{t+1}\right)dt$ [partial fractions] $= \ln|2t+1| - \ln|t+1| + C$

7. $\int_0^{\pi/2} \sin^3\theta\,\cos^2\theta\,d\theta = \int_0^{\pi/2}(1-\cos^2\theta)\cos^2\theta\,\sin\theta\,d\theta = \int_1^0 (1-u^2)u^2\,(-du)$ $\quad\left[\begin{array}{l} u = \cos\theta, \\ du = -\sin\theta\,d\theta \end{array}\right]$

$= \int_0^1 (u^2-u^4)\,du = \left[\frac{1}{3}u^3 - \frac{1}{5}u^5\right]_0^1 = \left(\frac{1}{3}-\frac{1}{5}\right) - 0 = \frac{2}{15}$

9. Let $u = \ln t$, $du = dt/t$. Then $\displaystyle\int \frac{\sin(\ln t)}{t}\,dt = \int \sin u\,du = -\cos u + C = -\cos(\ln t) + C$.

11. Let $x = \sec\theta$. Then

$$\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx = \int_0^{\pi/3} \frac{\tan\theta}{\sec\theta}\,\sec\theta\,\tan\theta\,d\theta = \int_0^{\pi/3} \tan^2\theta\,d\theta = \int_0^{\pi/3}(\sec^2\theta - 1)\,d\theta = \left[\tan\theta - \theta\right]_0^{\pi/3} = \sqrt{3} - \frac{\pi}{3}.$$

13. Let $w = \sqrt[3]{x}$. Then $w^3 = x$ and $3w^2\,dw = dx$, so $\int e^{\sqrt[3]{x}}\,dx = \int e^w \cdot 3w^2\,dw = 3I$. To evaluate I, let $u = w^2$, $dv = e^w\,dw \Rightarrow du = 2w\,dw$, $v = e^w$, so $I = \int w^2 e^w\,dw = w^2e^w - \int 2we^w\,dw$. Now let $U = w$, $dV = e^w\,dw \Rightarrow dU = dw$, $V = e^w$. Thus, $I = w^2e^w - 2\left[we^w - \int e^w\,dw\right] = w^2e^w - 2we^w + 2e^w + C_1$, and hence

$3I = 3e^w(w^2-2w+2) + C = 3e^{\sqrt[3]{x}}(x^{2/3} - 2x^{1/3} + 2) + C$.

15.
$\displaystyle\frac{x-1}{x^2+2x} = \frac{x-1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \Rightarrow x-1 = A(x+2)+Bx$. Set $x=-2$ to get $-3 = -2B$, so $B = \frac{3}{2}$. Set $x = 0$ to get $-1 = 2A$, so $A = -\frac{1}{2}$. Thus, $\displaystyle\int \frac{x-1}{x^2+2x}\,dx = \int\left(\frac{-\frac{1}{2}}{x} + \frac{\frac{3}{2}}{x+2}\right)dx = -\frac{1}{2}\ln|x| + \frac{3}{2}\ln|x+2| + C$.

17. Integrate by parts with $u = x$, $dv = \sec x\,\tan x\,dx \Rightarrow du = dx$, $v = \sec x$:

$\int x\sec x\,\tan x\,dx = x\sec x - \int \sec x\,dx \overset{14}{=} x\sec x - \ln|\sec x + \tan x| + C$.

19. $\displaystyle\int \frac{x+1}{9x^2+6x+5}\,dx = \int \frac{x+1}{(9x^2+6x+1)+4}\,dx = \int \frac{x+1}{(3x+1)^2+4}\,dx \qquad \begin{bmatrix} u = 3x+1, \\ du = 3\,dx \end{bmatrix}$

$$= \int \frac{\left[\frac{1}{3}(u-1)\right]+1}{u^2+4}\left(\frac{1}{3}\,du\right) = \frac{1}{3}\cdot\frac{1}{3}\int \frac{(u-1)+3}{u^2+4}\,du$$

$$= \frac{1}{9}\int \frac{u}{u^2+4}\,du + \frac{1}{9}\int \frac{2}{u^2+2^2}\,du = \frac{1}{9}\cdot\frac{1}{2}\ln(u^2+4) + \frac{2}{9}\cdot\frac{1}{2}\tan^{-1}\left(\frac{1}{2}u\right) + C$$

$$= \tfrac{1}{18}\ln(9x^2+6x+5) + \tfrac{1}{9}\tan^{-1}\left[\tfrac{1}{2}(3x+1)\right] + C$$

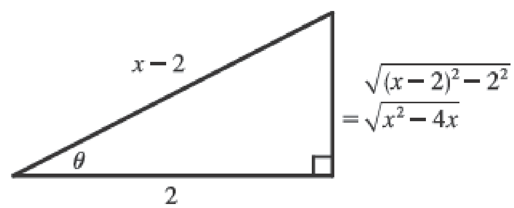
21. $\displaystyle\int \frac{dx}{\sqrt{x^2-4x}} = \int \frac{dx}{\sqrt{(x^2-4x+4)-4}} = \int \frac{dx}{\sqrt{(x-2)^2-2^2}}$

$$= \int \frac{2\sec\theta\,\tan\theta\,d\theta}{2\tan\theta} \qquad \begin{bmatrix} x-2 = 2\sec\theta, \\ dx = 2\sec\theta\,\tan\theta\,d\theta \end{bmatrix}$$

$$= \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C_1$$

$$= \ln\left|\frac{x-2}{2} + \frac{\sqrt{x^2-4x}}{2}\right| + C_1$$

$$= \ln\left|x-2+\sqrt{x^2-4x}\,\right| + C, \text{ where } C = C_1 - \ln 2$$

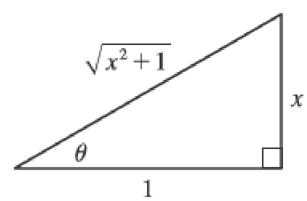


23. Let $x = \tan\theta$, so that $dx = \sec^2\theta\,d\theta$. Then

$$\int \frac{dx}{x\sqrt{x^2+1}} = \int \frac{\sec^2\theta\,d\theta}{\tan\theta\,\sec\theta} = \int \frac{\sec\theta}{\tan\theta}\,d\theta$$

$$= \int \csc\theta\,d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$= \ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| + C = \ln\left|\frac{\sqrt{x^2+1}-1}{x}\right| + C$$



25. $\dfrac{3x^3-x^2+6x-4}{(x^2+1)(x^2+2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+2} \quad\Rightarrow\quad 3x^3-x^2+6x-4 = (Ax+B)(x^2+2)+(Cx+D)(x^2+1).$

Equating the coefficients gives $A+C=3$, $B+D=-1$, $2A+C=6$, and $2B+D=-4 \quad\Rightarrow$

$A=3$, $C=0$, $B=-3$, and $D=2$. Now

$$\int \frac{3x^3-x^2+6x-4}{(x^2+1)(x^2+2)}\,dx = 3\int \frac{x-1}{x^2+1}\,dx + 2\int \frac{dx}{x^2+2} = \frac{3}{2}\ln(x^2+1) - 3\tan^{-1}x + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C.$$

27. $\int_0^{\pi/2}\cos^3 x\,\sin 2x\,dx = \int_0^{\pi/2}\cos^3 x\,(2\sin x\,\cos x)\,dx = \int_0^{\pi/2} 2\cos^4 x\,\sin x\,dx = \left[-\frac{2}{5}\cos^5 x\right]_0^{\pi/2} = \frac{2}{5}$

29. The integrand is an odd function, so $\displaystyle\int_{-3}^{3}\frac{x}{1+|x|}\,dx = 0$ [by 4.5.6(b)].

31. Let $u = \sqrt{e^x - 1}$. Then $u^2 = e^x - 1$ and $2u\,du = e^x\,dx$. Also, $e^x + 8 = u^2 + 9$. Thus,

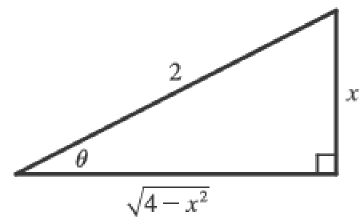
$$\int_0^{\ln 10} \frac{e^x\sqrt{e^x-1}}{e^x+8}\,dx = \int_0^3 \frac{u \cdot 2u\,du}{u^2+9} = 2\int_0^3 \frac{u^2}{u^2+9}\,du = 2\int_0^3 \left(1 - \frac{9}{u^2+9}\right)du$$

$$= 2\left[u - \frac{9}{3}\tan^{-1}\left(\frac{u}{3}\right)\right]_0^3 = 2\left[(3 - 3\tan^{-1}1) - 0\right] = 2\left(3 - 3\cdot\frac{\pi}{4}\right) = 6 - \frac{3\pi}{2}$$

33. Let $x = 2\sin\theta \quad\Rightarrow\quad \left(4 - x^2\right)^{3/2} = (2\cos\theta)^3$, $dx = 2\cos\theta\,d\theta$, so

$$\int \frac{x^2}{(4-x^2)^{3/2}}\,dx = \int \frac{4\sin^2\theta}{8\cos^3\theta}\,2\cos\theta\,d\theta = \int \tan^2\theta\,d\theta = \int \left(\sec^2\theta - 1\right)d\theta$$

$$= \tan\theta - \theta + C = \frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C$$



35. $$\int \frac{1}{\sqrt{x + x^{3/2}}}\,dx = \int \frac{dx}{\sqrt{x\,(1+\sqrt{x})}} = \int \frac{dx}{\sqrt{x}\sqrt{1+\sqrt{x}}} \quad \begin{bmatrix} u = 1+\sqrt{x}, \\ du = \dfrac{dx}{2\sqrt{x}} \end{bmatrix} \quad = \int \frac{2\,du}{\sqrt{u}} = \int 2u^{-1/2}\,du$$

$$= 4\sqrt{u} + C = 4\sqrt{1+\sqrt{x}} + C$$

37. $\int(\cos x + \sin x)^2\cos 2x\,dx = \int\left(\cos^2 x + 2\sin x\cos x + \sin^2 x\right)\cos 2x\,dx = \int(1 + \sin 2x)\cos 2x\,dx$

$$= \int\cos 2x\,dx + \tfrac{1}{2}\int\sin 4x\,dx = \tfrac{1}{2}\sin 2x - \tfrac{1}{8}\cos 4x + C$$

Or: $\int(\cos x + \sin x)^2\cos 2x\,dx = \int(\cos x + \sin x)^2(\cos^2 x - \sin^2 x)\,dx$

$$= \int(\cos x + \sin x)^3(\cos x - \sin x)\,dx = \tfrac{1}{4}(\cos x + \sin x)^4 + C_1$$

39. We'll integrate $I = \displaystyle\int \frac{xe^{2x}}{(1+2x)^2}\,dx$ by parts with $u = xe^{2x}$ and $dv = \dfrac{dx}{(1+2x)^2}$. Then $du = (x\cdot 2e^{2x} + e^{2x}\cdot 1)\,dx$

and $v = -\dfrac{1}{2}\cdot\dfrac{1}{1+2x}$, so

$$I = -\frac{1}{2}\cdot\frac{xe^{2x}}{1+2x} - \int\left[-\frac{1}{2}\cdot\frac{e^{2x}(2x+1)}{1+2x}\right]dx = -\frac{xe^{2x}}{4x+2} + \frac{1}{2}\cdot\frac{1}{2}e^{2x} + C = e^{2x}\left(\frac{1}{4} - \frac{x}{4x+2}\right) + C$$

Thus, $\displaystyle\int_0^{1/2}\frac{xe^{2x}}{(1+2x)^2}\,dx = \left[e^{2x}\left(\frac{1}{4} - \frac{x}{4x+2}\right)\right]_0^{1/2} = e\left(\frac{1}{4} - \frac{1}{8}\right) - 1\left(\frac{1}{4} - 0\right) = \frac{1}{8}e - \frac{1}{4}.$

41. $$\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty}\int_1^t \tfrac{1}{2}(2x+1)^{-3}\,2\,dx = \lim_{t\to\infty}\left[-\frac{1}{4(2x+1)^2}\right]_1^t$$

$$= -\frac{1}{4}\lim_{t\to\infty}\left[\frac{1}{(2t+1)^2} - \frac{1}{9}\right] = -\frac{1}{4}\left(0 - \frac{1}{9}\right) = \frac{1}{36}$$

43. $\int \frac{dx}{x \ln x} \quad \begin{bmatrix} u = \ln x, \\ du = dx/x \end{bmatrix} \quad = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C$, so

$$\int_2^{\infty} \frac{dx}{x \ln x} = \lim_{t \to \infty} \int_2^t \frac{dx}{x \ln x} = \lim_{t \to \infty} \Big[\ln|\ln x|\Big]_2^t = \lim_{t \to \infty} [\ln(\ln t) - \ln(\ln 2)] = \infty, \text{ so the integral is divergent.}$$

45. $\int_0^4 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{t \to 0^+} \int_t^4 \frac{\ln x}{\sqrt{x}}\,dx \overset{*}{=} \lim_{t \to 0^+} \Big[2\sqrt{x}\,\ln x - 4\sqrt{x}\Big]_t^4$

$$= \lim_{t \to 0^+} \left[(2 \cdot 2\ln 4 - 4 \cdot 2) - \left(2\sqrt{t}\,\ln t - 4\sqrt{t}\right)\right] \overset{**}{=} (4\ln 4 - 8) - (0 - 0) = 4\ln 4 - 8$$

$(*)$ Let $u = \ln x$, $dv = \frac{1}{\sqrt{x}}\,dx \quad \Rightarrow \quad du = \frac{1}{x}\,dx$, $v = 2\sqrt{x}$. Then

$$\int \frac{\ln x}{\sqrt{x}}\,dx = 2\sqrt{x}\,\ln x - 2\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\,\ln x - 4\sqrt{x} + C$$

$(**)$ $\lim_{t \to 0^+} \left(2\sqrt{t}\,\ln t\right) = \lim_{t \to 0^+} \frac{2\ln t}{t^{-1/2}} \overset{H}{=} \lim_{t \to 0^+} \frac{2/t}{-\frac{1}{2}t^{-3/2}} = \lim_{t \to 0^+} \left(-4\sqrt{t}\right) = 0$

47. $\int_0^1 \frac{x-1}{\sqrt{x}}\,dx = \lim_{t \to 0^+} \int_t^1 \left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right)dx = \lim_{t \to 0^+} \int_t^1 \left(x^{1/2} - x^{-1/2}\right)dx = \lim_{t \to 0^+} \left[\tfrac{2}{3}x^{3/2} - 2x^{1/2}\right]_t^1$

$$= \lim_{t \to 0^+} \left[\left(\tfrac{2}{3} - 2\right) - \left(\tfrac{2}{3}t^{3/2} - 2t^{1/2}\right)\right] = -\tfrac{4}{3} - 0 = -\tfrac{4}{3}$$

49. Let $u = 2x + 1$. Then

$$\int_{-\infty}^{\infty} \frac{dx}{4x^2 + 4x + 5} = \int_{-\infty}^{\infty} \frac{\frac{1}{2}\,du}{u^2 + 4} = \tfrac{1}{2}\int_{-\infty}^{0} \frac{du}{u^2 + 4} + \tfrac{1}{2}\int_0^{\infty} \frac{du}{u^2 + 4}$$

$$= \tfrac{1}{2}\lim_{t \to -\infty} \left[\tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}u\right)\right]_t^0 + \tfrac{1}{2}\lim_{t \to \infty} \left[\tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}u\right)\right]_0^t = \tfrac{1}{4}\left[0 - \left(-\tfrac{\pi}{2}\right)\right] + \tfrac{1}{4}\left[\tfrac{\pi}{2} - 0\right] = \tfrac{\pi}{4}.$$