3. $y = \sin x \quad \Rightarrow \quad dy/dx = \cos x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \cos^2 x$. So $L = \int_0^\pi \sqrt{1 + \cos^2 x}\, dx \approx 3.8202$.

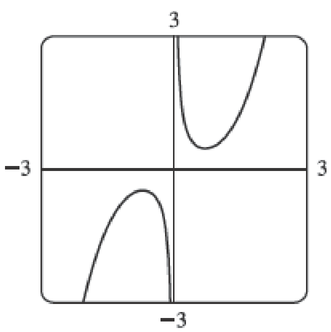
5. $x = \sqrt{y} - y \quad \Rightarrow \quad dx/dy = 1/(2\sqrt{y}) - 1 \quad \Rightarrow \quad 1 + (dx/dy)^2 = \left(\dfrac{1}{2\sqrt{y}} - 1\right)^2$.

So $L = \displaystyle\int_1^4 \sqrt{1 + \left(\frac{1}{2\sqrt{y}} - 1\right)^2}\, dy \approx 3.6095$.

7. $y = 1 + 6x^{3/2} \quad \Rightarrow \quad dy/dx = 9x^{1/2} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + 81x$. So

$$L = \int_0^1 \sqrt{1 + 81x}\, dx = \int_1^{82} u^{1/2}\left(\tfrac{1}{81}\, du\right) \quad \begin{bmatrix} u = 1 + 81x, \\ du = 81\, dx \end{bmatrix} \quad = \tfrac{1}{81} \cdot \tfrac{2}{3}\left[u^{3/2}\right]_1^{82} = \tfrac{2}{243}\left(82\sqrt{82} - 1\right)$$

9.



$$y = \frac{x^3}{3} + \frac{1}{4x} \quad \Rightarrow \quad y' = x^2 - \frac{1}{4x^2} \quad \Rightarrow$$

$$1 + (y')^2 = 1 + \left(x^4 - \frac{1}{2} + \frac{1}{16x^4}\right) = x^4 + \frac{1}{2} + \frac{1}{16x^4} = \left(x^2 + \frac{1}{4x^2}\right)^2. \text{ So}$$

$$L = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 \left| x^2 + \frac{1}{4x^2} \right| dx = \int_1^2 \left(x^2 + \frac{1}{4x^2}\right) dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{4x}\right]_1^2 = \left(\frac{8}{3} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{7}{3} + \frac{1}{8} = \frac{59}{24}$$

11. $x = \frac{1}{3}\sqrt{y}\,(y - 3) = \frac{1}{3}y^{3/2} - y^{1/2} \quad \Rightarrow \quad dx/dy = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2} \quad \Rightarrow$

$$1 + (dx/dy)^2 = 1 + \tfrac{1}{4}y - \tfrac{1}{2} + \tfrac{1}{4}y^{-1} = \tfrac{1}{4}y + \tfrac{1}{2} + \tfrac{1}{4}y^{-1} = \left(\tfrac{1}{2}y^{1/2} + \tfrac{1}{2}y^{-1/2}\right)^2. \text{ So}$$

$$L = \int_1^9 \left(\tfrac{1}{2}y^{1/2} + \tfrac{1}{2}y^{-1/2}\right) dy = \tfrac{1}{2}\left[\tfrac{2}{3}y^{3/2} + 2y^{1/2}\right]_1^9 = \tfrac{1}{2}\left[\left(\tfrac{2}{3} \cdot 27 + 2 \cdot 3\right) - \left(\tfrac{2}{3} \cdot 1 + 2 \cdot 1\right)\right]$$

$$= \tfrac{1}{2}\left(24 - \tfrac{8}{3}\right) = \tfrac{1}{2}\left(\tfrac{64}{3}\right) = \tfrac{32}{3}.$$

15.

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x \quad \Rightarrow \quad y' = \frac{1}{2}x - \frac{1}{2x} \quad \Rightarrow \quad 1 + (y')^2 = 1 + \left(\frac{1}{4}x^2 - \frac{1}{2} + \frac{1}{4x^2}\right) = \frac{1}{4}x^2 + \frac{1}{2} + \frac{1}{4x^2} = \left(\frac{1}{2}x + \frac{1}{2x}\right)^2.$$

So

$$L = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 \left|\frac{1}{2}x + \frac{1}{2x}\right| dx = \int_1^2 \left(\frac{1}{2}x + \frac{1}{2x}\right) dx$$

$$= \left[\frac{1}{4}x^2 + \frac{1}{2}\ln|x|\right]_1^2 = \left(1 + \frac{1}{2}\ln 2\right) - \left(\frac{1}{4} + 0\right) = \frac{3}{4} + \frac{1}{2}\ln 2$$

17. $y = \ln(1 - x^2) \quad \Rightarrow \quad y' = \dfrac{1}{1 - x^2} \cdot (-2x) \quad \Rightarrow$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4x^2}{(1 - x^2)^2} = \frac{1 - 2x^2 + x^4 + 4x^2}{(1 - x^2)^2} = \frac{1 + 2x^2 + x^4}{(1 - x^2)^2} = \frac{(1 + x^2)^2}{(1 - x^2)^2} \quad \Rightarrow$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{\left(\frac{1 + x^2}{1 - x^2}\right)^2} = \frac{1 + x^2}{1 - x^2} = -1 + \frac{2}{1 - x^2} \quad \text{[by division]} \quad = -1 + \frac{1}{1 + x} + \frac{1}{1 - x} \quad \text{[partial fractions]}.$$

So $L = \displaystyle\int_0^{1/2} \left(-1 + \frac{1}{1 + x} + \frac{1}{1 - x}\right) dx = \Big[-x + \ln|1 + x| - \ln|1 - x|\Big]_0^{1/2} = \left(-\tfrac{1}{2} + \ln\tfrac{3}{2} - \ln\tfrac{1}{2}\right) - 0 = \ln 3 - \tfrac{1}{2}$.